Ramsey-type results for the hypercube

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Abstract

We consider the question of existence of monochromatic cycles for edge colorings of the hypercube, raised by Fan Chung in 1992. She proved that for any fixed $k \geq 2$ and an *even* $l \geq 4$, for a sufficiently large hypercube, any k-coloring of the edges contains a monochromatic cycle of length $2l$. On the other hand, there is a 2-coloring of any hypercube which avoids monochromatic cycles of length 4, and a 3-coloring which avoids monochromatic cycles of length 6. Fan Chung asked what happens for *odd* $l \geq 5$, i.e. whether it is possible to avoid monochromatic cycles of length $10, 14, 18, \ldots$

We answer this question by proving that for any fixed $k \geq 2$ and $l \geq 5$, any k-coloring of a sufficiently large hypercube contains a monochromatic cycle of length $2l$. More generally, we provide a characterization of all subgraphs of the hypercube with this Ramsey property. In addition, we show the existence of subgraphs H_k such that for a sufficiently large hypercube, any k-coloring of the edges contains a monochromatic copy of H_k but this is not the case for all $(k+1)$-colorings.

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