APPLIED MATHEMATICS COLLOQUIUM

TOWARDS COMBINATORIAL PRECONDITIONERS FOR FINITE-ELEMENTS PROBLEMS

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ABSTRACT:

The talk will present a general algebraic definition of finite-element matrices and a combinatorial model of such matrices called a rigidity graph. The rigidity graph, which we can construct in linear time, represents in an efficient way some aspects (but not all) of the rigidity of the structure that the matrix models. More specifically, if the rigidity graph of the matrix is connected, then the underlying structure is rigid (the converse is not always true; deciding whether a structure is rigid is hard). This allows us to build a preconditioner by simplifying the matrix, for example by dropping elements, while ensuring that the matrix and preconditioner have the same range and null space. I will also present a novel and physically-inspired method for simplifying finite-element matrices called Fretsaw preconditioning.

Both the rigidity graph and the fretsaw-preconditioning algorithm are purely combinatorial-algebraic; they do not rely on the underlying physics or on the discretization (but they do rely on the element matrices being semidefinite). In particular, we define the concept of rigidity in a combinatorial-algebraic way, but our definition coincides with the traditional definition for problems in linear elasticity and in electrical circuits.

The talk will also present several important open problems and conjectures: the problem of analyzing the condition number of matrix pencils constructed with our rigidity graphs; the challenge of finding new rigidity-graph-based combinatorial preconditioners; and a conjecture that generalizes a bound from spectral graph theory (Cheeger's bound) to finite-element matrices.

MONDAY, FEBRUARY 27, 2006
4:30 PM
Building 2, Room 105

Reception at 4:00 PM in Building 4, Room 174.
(Math Majors Lounge)

Applied Math Colloquium: http://www-math.mit.edu/amc/spring06
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