APPLIED MATHEMATICS COLLOQUIUM

DISPERSION OF MASS AND THE COMPLEXITY OF RANDOMIZED ALGORITHMS

SANTOSH VEMPALA
Massachusetts Institute of Technology

ABSTRACT:

How much can randomness help computation? Motivated by this question, we analyze a notion of dispersion and connect it to asymptotic convex geometry. Two of the most appealing conjectures in the latter field are (i) slicing (or isotropic constant) and (ii) variance. Together they imply that for a random point X from an isotropic convex body in n-space, the variance of $|X|^2$ is $O(n)$. We prove a reverse inequality: for any isotropic convex polytope with at most poly(n) facets, the variance of $|X|^2$ is AT LEAST $n/\log n$ (up to a constant). In fact, the lower bound holds for any polytope of unit volume with poly(n) facets contained in the ball of radius poly(n). It implies that in order for most of such a convex polytope to be contained between two concentric spheres, their radii have to differ by about $1/\sqrt{\log n}$; in contrast, most of a unit-volume ball lies in between two spheres whose radii differ by only about $1/\sqrt{n}$. This geometric dispersion leads to linear and quadratic lower bounds on the randomized complexity of some basic algorithmic problems.

This is joint work with Luis Rademacher.

FRIDAY, FEBRUARY 10, 2006
4:30 p.m.

Building 4, Room 237

Refreshments will be served at 4:00 PM in Room 4-174
(Math Majors Lounge).