APPLIED MATHEMATICS COLLOQUIUM

CRITICAL THRESHOLDS IN EULERIAN DYNAMICS

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ABSTRACT:

We are concerned with the questions of global regularity vs. finite time breakdown in Eulerian dynamics, $\mathbf{u}_t + \mathbf{u} \cdot \nabla_x \mathbf{u} = \nabla_x F$. The global behavior is dictated by the different models of the forcing $F = F(\mathbf{u}, \nabla \mathbf{u}, \ldots)$. To address these questions, we propose the notion Critical Threshold (CT), where a conditional finite time breakdown depends on whether the initial configuration crosses intrinsic critical surfaces which guarantee global existence. With the standard energy method approach one studies the growth of $\nabla_x \mathbf{u}$. Our approach is based on spectral dynamics, tracing the eigenvalues, $\lambda := \lambda(\nabla_x \mathbf{u})$, which determine the boundaries of CT surfaces in configuration space.

We demonstrate the CT phenomena with several prototype models. We begin with the n-dimensional restricted Euler equations, obtaining a surprising 4-dimensional global existence for a large set of sub-critical initial data. The second example consists of the corresponding n-dimensional restricted Euler-Poisson equations. Here we identify a set of $[n/2]$ spectral invariants, which lead to a remarkable characterization of two-dimensional sub-critical initial configurations with global smooth solutions. Finally, we show how the CT phenomenon associated with rotation prevents finite-time breakdown, which, in turn, yields a long-time regularity regime in the shallow-water equations. Our study reveals the critical dependence of the two-dimensional CT phenomenon on the initial spectral gap, $\lambda_2(0) - \lambda_1(0)$.

Monday, April 24, 2006
4:30 p.m.
Building 2, Room 105

Reception at 4:00 PM in Building 4, Room 174
(Math Majors Lounge)

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